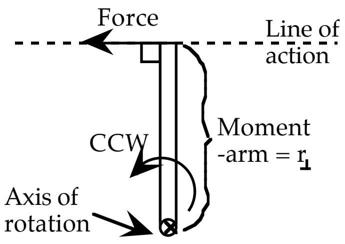
Chapter 11 Rotational Dynamics and Static Equilibrium

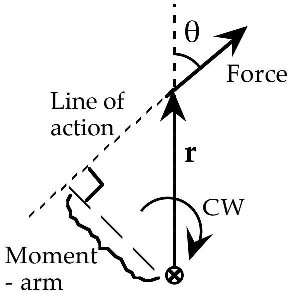
Section 11-1

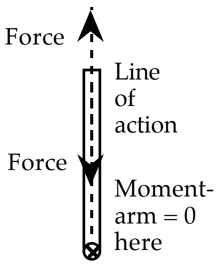
Torque Torque is defined as the product of a moment-arm and an applied force:

$$\tau = r_\perp F = r F \sin\theta = r F_\perp$$

- The force points along a direction that is called the line of action (see the examples below).
- The moment-arm ($r_\perp$) is measured as the perpendicular distance from the axis of rotation to the line of action (see the examples below).







- Torques (and their signs) are usually referred to in terms of the direction in which they would rotate the object: clockwise (CW) torques are considered (–) and counter-clockwise (CCW) torques are considered (+).
- An equivalent way of calculating torque is to take the distance (r) from the axis of rotation to the point where the force acts, and multiply that by the component of F ($F_\perp$) perpendicular to the direction of r.

Section 11-2

- **Rotational dynamic equation:**

$$\tau = I\alpha = \frac{\Delta L}{\Delta t}$$

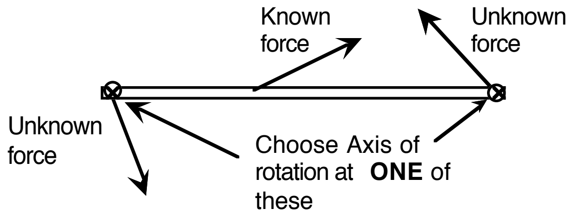
The dynamic equation ($\tau = I\alpha$) has a form that is similar to Newton's 2nd law, where (τ) behaves like force, (I) behaves like mass, and α behaves like acceleration. The $\Delta L/\Delta t$ extension is derived in section 11-6.

Section 11-3

Equilibrium conditions Both the sum of forces ($\Sigma \mathbf{F} = 0$ implies that linear acceleration $\mathbf{a} = 0$) and the sum of torques ($\Sigma \boldsymbol{\tau} = 0$ implies rotational acceleration $\alpha = 0$) must be zero in order for an object to be in equilibrium.

Problem solving hints on torque problems:

- The net torque acting on an object is determined by adding up all the clockwise and counterclockwise torques. Make sure that you include the correct – or + sign for each torque. If the object is in rotational equilibrium, then the sum of all the torques (positive and negative) must be zero. If the sum of torques is zero, then there must be at least two torques present: at least one positive and at least one negative.
- Determining what the moment-arms are in a particular torque problem requires that you first know where the axis of rotation is. If the object is in rotational equilibrium, you can choose any point you want for your axis of rotation, but usually the best point is one through which an unknown force's line of action passes (a moment-arm = 0 yields a torque = 0). From the example below, you can see that choosing one of the indicated points would result in a sum torques equation which has only one unknown force.

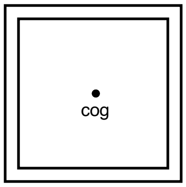


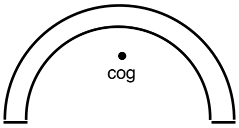
Section 11-4

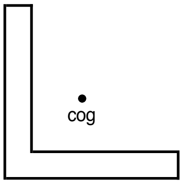
Center of gravity and center of mass The center of gravity is a convenient way to deal with the force due to gravity that acts on an object. An extended object (that isn't physically small enough to treat like a point) can be analyzed as if all of its weight *was* concentrated at one specific location. The center of gravity is determined the same way as the center of mass except each position is multiplied by the weight of the piece. The center of gravity (cog) location is then given by :

$$X_{cog} = \frac{\sum m_i\, g\, x_i}{M g} \qquad\qquad Y_{cog} = \frac{\sum m_i\, g\, y_i}{M g}$$

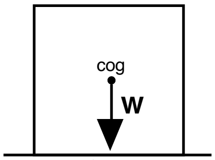
(Note that the weight of each piece "m g" is multiplied by the position of that piece– either "x" or "y".) In a uniform gravitational field (g = constant) these reduce to X_{cm} and Y_{cm}. For most objects the center of gravity will be located inside of the object, but this is not true for all objects. Consider the following two-dimensional examples:



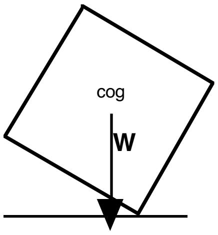




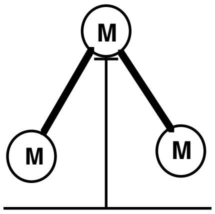
The location of an object's center of gravity and the orientation of the object determine it's stability. Generally speaking, objects supported from above their center of gravity tend to be more stable than those supported from below. Some examples:



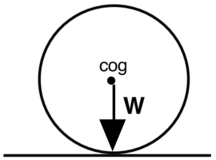
Stable (unless tilted, as in figure at right)



Unstable



Stable



Unstable (if pushed, will not return to original position)

Section 11-6

- Angular momentum is the momentum which an object possesses just because of its rotational motion. For an object small enough to treat like a point, angular momentum is L = mvr sin θ = p r sin θ. In general, we write: $\vec{\mathbf{L}} = I\vec{\boldsymbol{\omega}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$. (Note that this "looks" like $\vec{\mathbf{p}} = m\vec{v}$.)

Section 11-7

- The conservation of angular momentum is derived from the rotational form of Newton's 2nd law, where the sum of external torques is set equal to zero:

  $$0 = \boldsymbol{\tau} = I\alpha = I\,\frac{\Delta\omega}{\Delta t} = \frac{\Delta(I\,\omega)}{\Delta t} = \frac{\Delta L}{\Delta t}$$

  Thus, if $\tau = 0$, then $\Delta L = 0$, and L must be conserved (i.e. $L_i = L_f$).

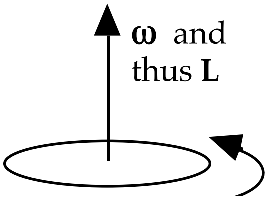
Section 11-8

- A torque that produces an angular displacement also produces a linear displacement ($\Delta x = R\Delta\theta$). Thus, a torque can do work: $W = \tau\,\Delta\theta$.

Section 11-9

- Vectors for rotational quantities follow a right hand rule: curl your right hand fingers in the direction of the rotational quantity (Δθ, ΔL) and your thumb points in the direction of the vector. remember that L and ΔL are not necessarily in the same direction.

At right , note that **ω** points in the same direction as **Δθ**, i.e. if Δθ is CCW, then ω will be CCW also.



At right , note that **τ** points in the same direction as **ΔL**, i.e. if ΔL is CCW, then τ will be CCW also.