Chapter 9 Linear Momentum and Collisions

Section 9-1

Momentum is a measure of an object's mass *and* state of motion: $\vec{\mathbf{p}} = m\vec{v}$.

- An object at rest has no momentum ($\vec{v} = 0$).
- Momentum is a vector. In general you will have to keep track of both x and y components of momentum. In this text we will mostly be concerned with one dimensional applications of momentum.

Section 9-2

- Newton's 2nd law is more correctly expressed in terms of momentum: $\Sigma \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$. In the special case where the mass of an object remains constant, this reduces to: $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

Section 9-3

- Impulse ($\vec{\mathbf{I}} = \vec{\mathbf{F}}_{av}\Delta t = \Delta\vec{\mathbf{p}}$) is given by the change in momentum that an object experiences. The force that appears in the formula is the average force that acts to change momentum $\Delta\vec{\mathbf{p}}$ over a time Δt. The average force is used in place of the actual force which can vary rapidly with time.

Section 9-4

- Conservation of momentum depends on external forces acting on a system summing to zero. If the sum of external forces acting on a system is not zero, then momentum is not conserved for that system.
- Internal forces in a system always sum to zero within the system, so only the absence of external forces is important in determining momentum conservation.
- Conservation of momentum applied to two interacting objects yields:

  $$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

  To solve this equation requires knowing 5 of the 6 different quantities (2 masses, 4 vectors) that appear in the equation. In two dimensions, this represents two component equations so that you have 2 masses and 8 components (4 each, in x and y).

- As usual, make sure you understand what initial and final values are given in a problem. Be careful with + and – signs. Even if only the ratio of masses $\left(\text{e.g. } \frac{m_2}{m_1} = 2\right)$ or the ratio of velocities $\left(\text{e.g. } \frac{v_2}{v_1} = \frac{1}{2}\right)$ is given, that may be enough to solve for the remaining unknown.

There are three basic types of collisions that can be analyzed:

Elastic	**Inelastic**	**Completely inelastic**
Both momentum and K conserved, objects separate afterwards	Only momentum conserved, objects separate afterwards	Only momentum conserved, but the objects stick together afterwards

Being able to identify which type of collision you are dealing with is an important skill. Remember that if the objects stick together, it must be a completely inelastic collision. If the objects separate after the collision and the velocity of one object is given, you can check that velocity for agreement with what is predicted by the elastic equations. E.g., use $v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2\, m_2}{m_1 + m_2}\right) v_2$ to see if the collision is elastic or not.

Section 9-5

Completely Inelastic Collisions Objects stick together after collision so that in one dimension, the final momentum is equal to the total mass times the final velocity:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

(In two dimensions, there would be two separate component equations in x and y.)

Section 9-6

Elastic Collisions As the example on page 255 points out, you will need to use both the momentum conservation equation (both x and y) and kinetic energy conservation in order to solve for the unknowns.

Section 9-7

Center of Mass This is the point where a system (one or more objects) can be balanced in a uniform gravitational field (this is somewhat of a preview for section 11-4). In two dimensions, the following expressions describe the location of the center of mass and its velocity and acceleration:

$$X_{cm} = \frac{\sum m_i x_i}{M} \qquad Y_{cm} = \frac{\sum m_i y_i}{M}$$

$$\vec{\mathbf{V}}_{cm} = \frac{\sum m_i \vec{\mathbf{v}}_i}{M} = \frac{\vec{\mathbf{p}}_{total}}{M} \qquad \vec{\mathbf{A}}_{cm} = \frac{\sum m_i \vec{\mathbf{a}}_i}{M} = \frac{\vec{\mathbf{F}}_{net}}{M}$$