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Introduction to Fractional Calculus and a Finite Element Formulation for Fractional Variational Problem

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Fractional Calculus has recently been applied in various areas of engineering, science, finance, applied mathematics, and bio engineering. However, many researchers remain unaware of this field. They often ask: What is a fractional derivative? Is this a new field or an old field? Are there applications of fractional derivatives? What are those applications? In this talk, several definitions of fractional derivatives will be introduced. Examples from different engineering fields will be presented to demonstrate that fractional derivatives arise naturally in many applications. A fractional derivative based formulation will be presented for thermal analysis of a disk brake, and the analytical results will be compared with the experimental results. A fractional variational problem will be introduced, and it will be demonstrated that the fractional Euler Lagrange equation for this problem leads to a new class of fractional differential equations with forward and backward fractional derivatives. A finite element formulation and numerical results will be presented to solve a fractional variational problem. Finally, possible directions for future research will be discussed.

Recessive Solutions of Nonoscillatory Halflinear Dynamic Equations

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We consider recessive solutions for the nonoscillatory halflinear dynamic equation

$$[a(t)\Phi(x^\Delta(t))]^\Delta + b(t)\Phi(x^\sigma(t)) = 0,$$

where $\Phi(u) = |u|^{p-2}u$, and a, b are positive rd-continuous functions on a time scale.

Global Existence and Blow Up for Higher Order Boussinesq Systems

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In this paper, we deal with higher order Boussinesq systems of equations that has been presented by Bona, Chen, Saut in [1]. We show that the solutions of these systems with a nonlinear power $\alpha > 1$ are global and decay in time for small initial data. We show also that they blow up in finite time.

Multi-point Boundary Value Problems of Fractional Order

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We are concerned with multi-point boundary value problems of fractional order with the Riemann-Liouville and Caputo fractional derivatives. The existence results are obtained using the Schauder fixed point theorem.

First Order Compatibility of the Cubic Schrodinger Equation

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In this talk we consider the compatibility between the nonlinear Cubic Schrodinger equation and a general quasilinear system of partial differential equations (PDEs). Recently it has been proven that the compatibility between PDEs and their associated invariant surface condition recovers the nonclassical method of symmetry reduction. This can be seen as a generalization of that method. Here we show that two classes of compatible equation exist. The first recovers the results obtainable by the classical symmetry method whereas the second is an entirely new class of compatible equation. We solve this in conjunction with the original system leading to a new reduction of the Cubic Schrodinger equation.

Practical Error Analysis of Numerical Solutions to the Huxley's Equation

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In this talk, I like to discuss, how long time error estimates are obtained using non-traditional methods for the Huxley's equation

$$u_t - u_{xx} = u(1 - u)(u - a) \quad \text{for } 0 < a < 1/2.$$

Traditional methods for analyzing exact error propagation depends on the stability of the numerical method employed. Whereas, in this talk the analysis of the exact error propagation uses evolving attractors and only depends on the stability of the dynamical system. The use of the smoothing indicator yields *a posteriori* estimates on the numerical error instead of *a priori* estimates.

On Wave Equations with Interior and Boundary Interactions Between Supercritical Sources and Dampings

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The model considered is the wave equation with Neumann boundary conditions and interior and boundary interactions of supercritical nonlinear sources and dampings. Our aim is to discuss the wellposedness of the system on finite energy space. Since Lopatinski condition fails to hold, the analysis of the nonlinearities supported on the boundary, within the framework of weak solutions, is a subtle issue and involves strong interaction between the source and the damping. We will provide positive answers to the questions of local existence and uniqueness of weak solutions and moreover give complete and sharp description of parameters corresponding to global existence and blow-up of solutions in finite time.

Logistic Dynamic Equations in Population Models

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We give a brief introduction to the theory of dynamic equations on time scales. Then we proceed to present and verify the Cushing-Henson conjectures on time scales. The central part of these conjectures asserts that based on a model using the dynamic Beverton-Holt equation, a periodic environment is deleterious for the population. The proof technique is as follows: First, the Beverton-Holt equation is identified as a logistic dynamic equation. The usual substitution transforms this equation into a linear equation. Then the proof is completed using a recently established dynamic version of the generalized Jensen inequality. If time permits, we also will consider the case of harvesting with constant effort and will maximize the seasonal sustainable yield to find the optimal harvesting policy and the optimal population level. The main objects in this talk are logistic differential equations, logistic difference equations, and their unified counterparts, logistic dynamic equations on time scales.

Model Equations for Waves and Some Applications to Problems in Geophysics

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After a very brief tour through the historical context, we explain how model equations for waves arise as approximation of the full water wave problem. Attention is then turned to some interesting applications of these model equations to issues arising in describing some of the large scale motions of the ocean and in coastal engineering contexts.

Sensitivity Analysis for MinMax Parameter Choice for a Nonlinear Cable-Mass System

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The goal of this work is to minimize the computational cost associated with the experimental determination of the critical parameter used in the design of the MinMax controller. To this end, the sensitivity of the controlled state to variations in the parameter is examined mathematically using continuous sensitivity equations. The sensitivity of the controllers performance, robustness and convergence with respect to the parameter is also studied. By analyzing these various sensitivities, it is hoped that a mathematically justified parameter can be determined.

Approximating Initial-Value Problems with Two-point Boundary-value Problems: BBM-equation

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The focus of the present study is the BBM equation which models unidirectional propagation of small amplitude long waves in dispersive media. This evolution equation has been used in both laboratory and field studies of water waves. The principal new result is an exact theory of convergence of the two-point boundary-value problem to the initial-value problem posed on an infinite stretch of the medium of propagation. In addition to their intrinsic interest, our results provide justification for the use of the two-point boundary-value problem in numerical studies of the initial-value problem posed on the entire line.

Mathematical Modeling for the effects of HER2 Overexpression in Breast Cancer

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Over-expression of the HER2 receptor due to the neu gene amplification contributes to the development of human breast cancers. We present a mathematical model to study the effects of HER2 over-expression on cell proliferation. This mathematical model comprises kinetic equations describing the cell surface

binding of EGF growth factor to EGFR and HER2 receptors, coupled to a model for the dependence of cell proliferation rate on growth factor receptors binding. The cell proliferation models enable us to simulate the proliferative behavior of the HER2-overexpressing cells with various HER2 and EGFR expression levels at various ligand concentrations. This model predicts a growth advantage associated with excess in cell surface HER2 receptors.

Uniqueness Implies Existence and Uniqueness Conditions for a Class of $(k + j)$ -Point Boundary Value Problems for n th Order Differential Equations

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For the n th order nonlinear differential equation, $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$, we consider uniqueness implies existence results for solutions satisfying certain $(k + j)$ -point boundary conditions, $1 \leq j \leq n - 1$, and $1 \leq k \leq n - j$. We define $(k; j)$ -point unique solvability in analogy to k -point disconjugacy and we show that $(n - j_0; j_0)$ -point unique solvability implies $(k; j)$ -point unique solvability for $1 \leq j \leq j_0$, and $1 \leq k \leq n - j$. This result is in analogy to n -point disconjugacy implies k -point disconjugacy, $2 \leq k \leq n - 1$.

The Behavior of a Population Under the Effects of the Propagation of a Disease with Fixed Infection Length

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In this paper we study the existence of oscillating solutions of models for diseases with infective period of fixed length. Such oscillations characterize the fact that the number of infected individuals can go up and down several time, contradicting the usual peak for epidemic disease.

Application of Richardson Extrapolation to the Numerical Solution of Partial Differential Equations

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Richardson extrapolation is an algebraic technique for increasing the order of accuracy of numerical schemes, both the numerical formula and results generated, by uniformly refining the discretization of the equations and considering the resulting error. This method has been applied to numerical differentiation, numerical integration, and numerical solutions to ordinary differential equations. Herein, Richardson extrapolation is applied to two-dimensional numerical solvers of partial differential equations. Previous work by the author in one dimension is reviewed, and the methodology developed is extended to two dimensional finite volume solvers on unstructured triangular meshes with specific application to the shallow water equations.

Control Parameter Determination for Low Order Controllers

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The H_∞ controller has received much research attention because of its robustness to disturbances and uncertainties since first being introduced by Zames. Rhee and Speyer later introduced the MinMax controller, which is a differential game approach to solving the H_∞ control problem. This talk will focus on the determination of the MinMax control parameter using a non-iterative methodology in the design of low order LQG balanced controllers. Preliminary results based on the comparison of performance, robustness, and convergence properties of low order MinMax controllers will be presented. The problem of interest in this discussion is a nonlinear cable-mass system.

A Population Model with Nonlinear Boundary Conditions and Constant Yield Harvesting

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We analyze the solutions of a population model with diffusion and strong Allee effect. In particular, we focus our study on a population that satisfies a certain nonlinear boundary condition and its survival when constant yield harvesting is introduced. In this presentation we will discuss results obtained in the one-dimensional case.

A Uniformly Convergent Hybrid Finite Difference Scheme for Two-Parameter Singularly Perturbed Parabolic Problem

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In the present work, a parameter-uniform numerical method is constructed for solving one-dimensional singularly perturbed parabolic problems with two small parameters modeling diffusion-convection-reaction processes on a rectangular domain. The solution of this class of problems may exhibit the exponential (or parabolic) boundary layers at both the left and right part of the lateral surface of the domain, depending on the size of the parameters. The asymptotic behaviour of the solution and its partial derivatives is given. A decomposition of the solution in its regular and singular parts has been used for the asymptotic analysis of the spatial derivatives. To approximate the solution we consider the implicit Euler method for time stepping on uniform mesh and a special hybrid monotone difference operator for spatial discretization on a special piecewise uniform Shishkin mesh. We prove that this scheme is uniformly convergent, with respect to both the singular perturbation parameters, having first-order convergence in temporal direction and almost second-order convergence in spatial direction, in the discrete maximum norm. Numerical experiments illustrate the order of convergence proved theoretically.

A Field Theory Two-body Problem in Acoustics

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An acoustic model is developed to illustrate a scheme for obtaining effective equations of motion for sources in field theories, where finite propagation speeds lead to radiation reaction and runaway solutions. The model is a hybrid system of PDEs for fluid pressure and velocity fields, coupled to ODEs for pistons that bound the fluid at each end. Elimination of the fields yields neutral functional differential equations for the pistons. Effective equations of motion are obtained by reduction to an inertial manifold, and may be approximated by expansion, truncation, and reduction to a slow manifold of a singular perturbation problem.

A Numerical Solver for Burgers' Equations in Two Dimensions

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In this presentation we introduce a numerical partial differential equation solver of the following system of Burgers' equations:

$$\partial u / \partial t = \lambda_1 (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) + u (\partial u / \partial x + \partial u / \partial y) + f_1(u, v) \partial v / \partial x + f_2(u, v) \partial v / \partial y$$

$$\partial v / \partial t = \lambda_2 (\partial^2 v / \partial x^2 + \partial^2 v / \partial y^2) + v (\partial v / \partial x + \partial v / \partial y) + g_1(u, v) \partial u / \partial x + g_2(u, v) \partial u / \partial y$$

where the λ_i s are constants that scale the diffusivity exhibited by the system, while the f_i s and g_i s are source terms. This code was tested on a decoupled case involving only the convective terms, a decoupled case with diffusion and a case where the system is coupled by setting the f_i s and g_i s to 1.

A Model for Transfer Phenomena in Structured Populations

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P-glycoprotein causes resistance to cancer chemotherapy as it is able to remove toxic substances from the cytoplasm of a cell. Recent experiments shows that cells rich in P-gp can transfer part of their P-gp to other cells and thereby may confer resistance. We propose a model for the transfer of a scalar quantity

in a structured population. We show the convergence of solutions to a Radon measure. We add to this model production of P-gp and random fluctuations and show the existence of a globally asymptotically stable steady state, if the rate of P-gp transfer is sufficiently small.

Multiplicity of Positive Solutions for an Even-order Nonhomogeneous Boundary Value Problem

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In this talk, we focus on the existence of multiple positive solutions for the $2n$ th order ordinary differential equation,

$$u^{(2n)} = \lambda h(t, u, u'', \dots, u^{(2(n-1))}), \quad t \in (0, 1),$$

satisfying the boundary conditions

$$u^{(2k)}(0) = 0, \quad k = 0, \dots, n - 1$$

$$u^{(2k+1)}(1) = (-1)^k a_k, \quad k = 0, \dots, n - 1$$

where $\lambda, a_0, \dots, a_{n-1} \geq 0$, with $\sum_{k=0}^{n-1} a_k > 0$, and $h : [0, 1] \times \prod_{i=0}^{n-1} (-1)^i [0, \infty) \rightarrow (-1)^n [0, \infty)$ is continuous. We transform the even order boundary value problem into a system of second order differential equations satisfying homogeneous right focal boundary conditions. Then, by applying the Guo-Krasnosel'skii Fixed Point Theorem several times, we show the existence of multiple positive solutions.

Traveling Waves and Their Stabilities for a Public Goods Games Model

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We study the traveling wave solutions of a competition model arising from public goods games. The existence of the traveling wave solutions is proved via monotone iteration of a pair of smooth upper- and lower-solutions. Such result implies that the altruistic behaviors of one species is the direct cause of its failure in the competition. We also derive an accurate value for the critical wave speed. The monotonicity and unique-ness of the traveling wave solution

are also investigated via Sliding Domian method. For traveling waves with non-critical wave speed, A new method of spectral analysis is employed to show their asymptotic stability.

Almost Linear Volterra Equations

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Boundedness and periodicity of certain Volterra type equations are considered. These equations are studied under a set of assumptions on the functions involved in the equations. The equations will be called almost linear when these assumptions hold.

About One Feature of the Property of Linear Systems of Differential Equations with Non-negative Coefficients.

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The families of morphisms of vector fibre bundle defined by the linear systems of differential equations with non-negative coefficients are considered. Author proved that the specified families of morphisms are not saturated (V. Million-schikov).

Non Conformal Perfect Fluid Plates in 5-Flat Form Using Similarity Transformations

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In view of renewed interest now a days, in the space time embedded in higher dimensional flat space. A set of six solutions for non-conformally flat perfect fluid plates is obtained using similarity transformations method by considering a five dimensional flat metric subjected to the plane symmetry. Most of the solutions so obtained are in implicit form. However some particular explicit cases are analyzed physically. Some of the fluid distributions are seen to be accelerating.

Higher-Order Compact Volume Differencing Schemes for Inhomogeneous Helmholtz Equations with High Wave Numbers

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Higher-order compact volume differencing schemes are developed and implemented for elliptic equations. Through constrained minimization of the local truncation error formulated from the conservative integral representation of the equation, control volume distribution of the source is used to account for the solution fluxes to produce the weights to describe the resulting schemes which are then optimized to improve consistency and stability.

Multivariate Taylor approximations for solution and the source about the center of grid-point centered control volumes and constrained by the derivatives of the approximate equation are utilized to mimic the integral representation of the equation to describe the formal error.

Numerical results and analysis of the schemes to demonstrate the effectiveness of the methodology for inhomogeneous Helmholtz equations will be presented.

A Numerical Solution of the Generalized Burger's-Huxley Equation Using Cubic B-Spline

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In this paper, numerical solutions of the nonlinear generalized Burger's-Huxley equation are obtained by a method based on collocation of cubic B-splines over finite elements. Applying the Von-Neumann stability analysis, the proposed method is shown to be conditionally stable. The accuracy of the presented method is demonstrated by test problems. The numerical results are found to be in good agreement with the exact solutions.

A Model for Submarine Avalanches Over Erodible Bed

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Submarine avalanches and landslides not only participate in the constant change of the sea floor but also represent a threat to the undersea inhabitants and human facilities. When studying submarine avalanches, one can not neglect the effect of erosion. Because the surface of the ocean bed is fluidized, erosion occurs more easily. We will present a one-dimensional Savage-Hutter model of geophysical mass flow to describe submarine avalanches which takes erosion into account.

Positive Solutions for Elliptic Equations Involving Nonlinearities with Falling Zeroes

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We study classes of boundary value problems involving the p -Laplacian operator and nonlinearities which have falling zeroes. We analyze the existence and multiplicity of positive solutions when a parameter is large. We use the method of sub-super solutions to establish our results.

Mild Solutions of an Initial Value Problem of Fractional Order

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In this paper, we obtain the existence and uniqueness of solutions to a fractional order initial value problem with the Caputo derivative. The inhomogeneous term satisfies the Caratheodory conditions with respect to a suitable Lebesgue space.

Modeling Glucose Mobilization and Uptake at a Molecular Level

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In this presentation, we propose a new mathematical control system for a simplified regulatory system of blood glucose by taking into account the dynamics of glucose and glycogen in liver and the dynamics of insulin and glucagon receptors at the molecular level. Numerical simulations show that the proposed feedback control system agrees approximately with published experimental data. Sensitivity analysis predicts that feedback control gains of insulin receptors and glucagon receptors are robust. Using the model, we develop a new formula to compute the insulin sensitivity. The formula shows that the insulin sensitivity depends on various parameters that determine the insulin influence on insulin-dependent glucose utilization and reflect the efficiency of binding of insulin to its receptors. Using Lyapunov indirect method, we prove that the new control system is input-output stable. The stability result provides theoretical evidence for the phenomenon that the blood glucose fluctuates within a narrow range in response to the exogenous glucose input from food. We also show that the regulatory system is controllable and observable. These structural system properties could explain why the glucose level can be regulated.

Bounded Solutions of Nonlinear Parabolic Systems with Nonlinear Boundary Conditions

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We consider coupled systems of two nonlinear parabolic equations on a domain which is unbounded in time (namely the entire real line) and bounded in space with nonlinear boundary conditions. We establish the existence of bounded solutions existing for all time by using a combination of a priori estimates, comparison techniques, interpolation inequalities, approximations, and embedding of function spaces. We illustrate the results with examples which include cooperative or competitive models in ecology.

MAC-Solution of Hertz Problem for an Elastic Plate

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Consider the linear partial differential equation with the delta-function on the right side. The particular solutions of this equation are called the Greens functions and they play an important role in the method of Greens functions to solve the correspondent boundary value problems. Let us consider the physical value of these Greens functions. Then, for example, the string and the beam problems in statics are one dimensional problems and represent the physically accepted finite solutions for displacements and for the stresses for the fixed boundary conditions. The two dimensional problem for membrane creates singularity in displacements which is not physical. The two dimensional problem for an elastic plate gives finite displacements but infinite stresses which are not physical. The Hertz problem is an elastic infinite plate on elastic support under a transverse force. This problem has finite displacements and nonphysical infinite stresses. The method of additional conditions (MAC) creates the non-singular solution on the base of the solution with singularity. MAC is not an asymptotic method or a method of regularization because for example they are applied only to the problems with existent solutions. MAC can be applied to the problems with nonexistent solutions. Strictly speaking the solution of Hertz problem does not exist because the Hertz solution includes the function with singularities in stresses. It is not too logically to accept the functions which give the singularities in stresses and to avoid the functions which give singularities in displacements. The MAC-solution for the Hertz problem for an elastic plate with finite stresses but using additional condition in form of line integral is obtained. The MAC-solution for the plate follows from the MAC-solution of the membrane.

On the Almost Periodic Solutions of Differential Equations on Hilbert Spaces

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For the linear differential equation $u'(t) = Au(t) + f(t)$, where A is an unbounded linear operator on a Hilbert space H and $f(t)$ is a function from R to H , we use the Fourier-Bohr series method to find the necessary and sufficient conditions, such that for each almost periodic function $f(t)$, there exists a unique almost periodic mild solution to the above equation.

Elliptic Equations with Nonlinear Boundary Conditions

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We will present existence and non-existence results for second order elliptic partial differential equations with nonlinear boundary conditions. The non-linearity on the boundary interacts in some sense with the eigenvalues of the Steklov spectrum. Our approach is based on a priori estimates, a variational approach and topological degree arguments. This is a joint work with Dr. Nsoki Mavinga of the University of Rochester, NY.

Completions of Implicitly Defined Linear Time Varying Vector Fields

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Differential algebraic equations (DAE), $F(x', x, t) = 0$, are mixed systems of differential and algebraic equations and occur naturally in many physical processes. Traditional ODE methods such as Backward differentiation and Runge-Kutta methods can be used to solve DAEs but they are limited to only lower index systems with special structure.

A popular method for solving more general DAEs is by obtaining an ODE whose solutions contains those of the DAE, which is called a completion. This can be done in an efficient manner by using the least squares technique. However, the additional dynamics can affect the numerical integration.

We have previously introduced two methods for stabilizing the completions of linear time invariant DAEs. In this talk we will present recent findings regarding the extension of these results to linear time varying systems.

The Regularity Problem for the Lamé System of Elastostatics on Curvilinear Polygons in Two Dimensions

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We establish sharp well-posedness results for the regularity problem for the Lamé system of elastostatics in the class of curvilinear polygons in two dimensions. The key technical ingredient is obtaining invertibility properties for the boundary version of the single layer potential operator S associated with the Lamé system acting from $L^p(\partial\Omega)$ onto $L_1^p(\partial\Omega)$, $1 < p < \infty$, whenever Ω is an infinite sector in two dimension of aperture $\theta \in (0, 2\pi)$. Our approach relies on Mellin transform techniques employed to analyze the spectrum of the operator $\partial_\tau S$ on $L^p(\partial\Omega)$, $1 < p < \infty$.

Exponential Analysis of Solutions of Functional Differential Equations with Unbounded Terms

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In this paper, we make use of non-negative definite Lyapunov functions and obtain sufficient conditions that guarantee the boundedness of all solutions of the system of functional differential equations

$$x'(t) = G(t, x(s); 0 \leq s \leq t) := G(t, x(\cdot)) \quad (1)$$

where $x \in \mathbb{R}^n$, $G : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a given nonlinear continuous function in t and x .

A stereotype of equation (1) is the Volterra Integrodifferential equation

$$x'(t) = h(t, x) + \int_0^t B(t, s)f(x(s))ds + g(t), \quad (2)$$

where $g(t)$ may be unbounded. The theory is illustrated with several examples.

Stability of Solitons for the KdV Equation in H^s , $-\frac{3}{4} < s < 0$

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We study the long-time stability of soliton solutions to the Korteweg-deVries equation. We consider solutions u to KdV with initial data in H^s , $-\frac{3}{4} < s < 0$, that are initially close in H^s norm to a soliton. We prove that the possible orbital instability of these ground states is at most polynomial in time.

On the Variational Characterization of Solutions to a Nonsmooth Cahn-Hilliard Equation

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Consider the functional

$$J(u) = \frac{\epsilon^2}{2} \int_0^1 |u'|^2 dx + \int_0^1 F(u) dx, u \in W^{1,2}(0,1),$$

where F is a *double-well potential* such as $F(u) = (1 - u^2)^2$. This functional represents the total free energy in models of phase transition. In this talk I describe how the complete list of critical points for J can be found and then classified as either minima or saddle points. Of particular interest will be some rather surprising continua of critical points that only appear when F loses its smoothness at $u = \pm 1$.

Two Level Finite Element Approximation of Navier-Stokes Equations with Nonlinear Subgrid-scale Artificial Viscosity

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In this talk, we will review the concept of two-level finite element approximation schemes for the Navier-Stokes equations. We will review the results of a recent article of ours, (J. Borggaard, T. Iliescu, H.K. Lee, J.P. Roop and H. Son, "A two-level discretization method for the Smagorinsky model," *Multiscale Modeling & Simulation*, **7(2)**, pp. 599-621, (2008)), in which finite element convergence estimates and scaling estimates and numerical results were proven for a two-level approximation scheme for the Smagorinsky model. At its core a

two-level approximation scheme allows for the same order of convergence at a fraction of the computational cost. We then introduce results from a new study which includes the possibility of varying the type of nonlinear subgridscale artificial viscosity model used and numerical results in three spatial dimensions. It is interesting to note that choosing the correct scaling coefficients leads to improved performance of the numerical algorithm in question.

Ornstein-Uhlenbeck Dynamic Equation and its Connection with the Vasicek Mean Reversion Model

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We introduce the Ornstein-Uhlenbeck dynamic equation $X^\Delta(t) = -\alpha X(t) + \beta W^\Delta(t)$, $X(t_0) = X_0$, where W is a Wiener process on \mathbb{T} and derive a representation for its solution. Results related to the expectation and variance of the stochastic process X and its connection with the Vasicek mean reversion model are presented.

Stability of the Diagonal Waves for Discretizations of the Advection Equation in Multidimensions

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We consider the advection equation in multidimensions discretized on a rectangular grid using spatial stencils of up to eight order of accuracy, and analyze the conditional stability of the diagonal waves (with respect to the grid lines). The time marching is carried out via either linear multistep or Runge-Kutta type methods. Compared to existing schemes, more favorable stability restrictions are found, and tested.

Multiple Positive Solutions for a Class of p-q-Laplacian Systems with Multiple Parameters and Combined Nonlinear Effects

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Consider the system

$$\begin{cases} -\Delta_p u &= \lambda_1 f(v) + \mu_1 h(u), \text{ in } \Omega \\ -\Delta_q v &= \lambda_2 g(u) + \mu_2 \gamma(v), \text{ in } \Omega \\ u = 0 = v, &\text{ on } \partial\Omega \end{cases}$$

where $\Delta_s z = \operatorname{div}(|\nabla z|^{s-2} \nabla z)$; $s > 1$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\mu_1 \geq 0$ and $\mu_2 \geq 0$ are parameters and Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$. For some classes of non-negative monotone functions f, g, h and γ which satisfy

$$\lim_{x \rightarrow \infty} \frac{f(M[g(x)]^{1/q-1})}{x^{p-1}} = 0, \quad \forall M > 0,$$

$\lim_{x \rightarrow \infty} \frac{h(x)}{x^{p-1}} = 0$ and $\lim_{x \rightarrow \infty} \frac{\gamma(x)}{x^{q-1}} = 0$, we discuss the existence of multiplicity of positive solutions for certain range of parameters $\lambda_1, \mu_1, \lambda_2$ and μ_2 . We use the method of sub- and super-solutions to establish our results.

Positive Solutions for Classes of n x n Nonlinear Elliptic Systems

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We study the positive solutions to classes of n x n reaction diffusion equations when the reactions satisfy a combined sublinear condition at infinity. We analyse positive as well as semipositone systems. We establish our results via the method of sub-super solutions.

Controlling the Motion of Charged Particles in a Vacuum Electromagnetic Field from Boundary

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We consider the problem of driving two non-relativistic charged particles in a bounded vacuum electromagnetic field to a same location by applying electromagnetic forces through the boundary of the domain. The dynamics of the particles is modeled by Maxwells equations coupled with the Lorentz force law and the problem is reduced to a boundary feedback control problem. Using the perturbed energy method, we design feedback controllers and prove that the particles under the designed control move to the origin exponentially. Our result may have potential applications in particle acceleration and nuclear fusion.

Lyapunov's Exponents of Linear Differential Equation of n-order as a Second Baire Class Functions of Continuous Parameter

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Lyapunov's exponents of one-parametrical family of linear differential equations of n-order as a functions of parameter are investigated.

Stabilization of Systems with Uncertain Parameters in a class of Structurally Stable Functions from Catastrophe Theory

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An approach to stabilization of dynamical systems with uncertain parameters is offered. As main advantage it provides the stability of the systems at uncertainly disturbed parameters. Finally the safe ranges of parameters are widened extremely. The method relies on using of nonlinear structurally stable functions from catastrophe theory as controllers. Theoretical part presents an analysis of designed nonlinear second-order control systems. As more important the integrators in series, canonical controllable form and Jordan forms are considered. The analysis resumes that due to added controllers systems become stable and insensitive to any disturbance of parameters. Experimental part presents MATLAB simulation of design of control systems of epidemic spread, aircrafts angular motion and submarine depth. The results of simulation confirm the efficiency of offered method of design.

Bifurcations in Steady State Solution of Damped Forced KdV Equation

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We consider the damped forced KdV equation. Using Asymptotic perturbation method bifurcations of steady state solution are discussed. (Preliminary report)

Mathematical Modeling of Blood Flow in the Stenotic Tapered Artery Under the Action of Periodic Body Acceleration

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Arterial stenosis is one of the most serious forms of arterial disease. The presence of a stenosis makes an artery work in the direction, which is opposite to the direction of a healthy one. Tapering is a significant aspect of mammalian arterial system. Due to the tapering, the oscillation amplitude of the wall shear stress is bigger than that of a non-tapered artery. The human body is often subjected to body acceleration or vibrations in many situations of day-to-day life while traveling or driving in Vehicles, Water ships, airplanes and in fast body movements in sports activities. A mathematical model is developed for the pulsatile flow of blood through a stenosed artery under the influence of externally imposed body acceleration. The blood is considered as a power law fluid. The artery is assumed to be having the mild stenosis with tapering. The governing equations are solved numerically by using a suitable finite difference scheme in order to obtain the velocity, fluid acceleration, wall shear stress and flow rate.

Numerical Simulation of Population Distributions in Two Dimensions

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Numerical partial differential equation solvers have been used to solve a variety of systems of differential equations, such as those from hydrodynamics and aeronautics. Here, we apply this technology to population modeling. We begin with the general population model for two species and formulate a system of partial differential equations, where the desirability regions and competition rates vary spatially. We calculate the eigensystem for the convective terms, develop an appropriate method for discretizing the diffusion terms, and implement the governing equations within an existing two-dimensional numerical solver. We consider a two-species competing model and a two species predator-prey model.

Super Central Configurations

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In this talk, we consider the inverse problem of central configurations of n -body problem. For a given $q = (q_1, q_2, \dots, q_n) \in (\mathbf{R}^d)^n$, let $S(q)$ be the admissible set of masses by

$$S(q) = \{m = (m_1, m_2, \dots, m_n) \mid m_i \in \mathbf{R}^+, q \text{ is a central configuration for } m\}.$$

We call $m = (m_1, m_2, \dots, m_n) \in S(q)$ and $m' = (m'_1, m'_2, \dots, m'_n) \in S(q)$ *equivalent* if $m = \alpha m'$ for some $\alpha \in \mathbf{R}^+$ and let $\tilde{S}(q)$ be the set of equivalent classes in $S(q)$. Denote by $\#\tilde{S}(q)$ the cardinal number of equivalent elements for any given configuration $q = (q_1, q_2, \dots, q_n) \in (\mathbf{R}^d)^n$. We call q a super central configuration if $\#\tilde{S}(q) \geq 2$. The possible values of $\#\tilde{S}(q)$ and the structure of the set $S(q)$ are investigated for $n \leq 4$. The existence of super central configuration for $n > 4$ is constructed. We prove that all central configurations are super central configurations for $n \leq 3$ and no convex four-body central configuration is a super central configuration. We also prove that a super central configuration has an unusual property: a super central configuration gives rise to a perverse solution when center of mass of the configuration is fixed.

Infinite Semipositone Problems

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We analyze the positive solutions to singular boundary value problems of the form

$$\begin{aligned} -\Delta u &= \lambda \frac{f(u)}{u^\alpha}; x \in \Omega \\ u &= 0; x \in \partial\Omega, \end{aligned}$$

where λ is a positive parameter, Ω is a bounded region in R^n , $n \geq 1$ with smooth boundary $\partial\Omega$, $\alpha \in (0, 1)$, Δ is the Laplacian operator and f is continuous with $f(0) < 0$. Note that $g(s) = \frac{f(s)}{s^\alpha} \rightarrow -\infty$ as $s \rightarrow 0^+$ (Infinite Semipositone case). We establish our results by the method of sub-super solution. We also establish extensions to the p -Laplacian case as well as to systems.

Divergence Free H(div) Finite Element Method for Incompressible Flow

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We derive and analyze a numerical formulation for the Stokes and Navier-Stokes equations which make use of H(div) elements. The finite element solutions feature exact satisfaction of incompressible constraint. In the formulation, we seek the velocity from the exactly divergence free subspace of H(div) elements. Therefore solving a saddle point problem can be avoided.

Global Dynamics of Diffusive Selkov Equations

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For the diffusive Selkov equations modeling cubic autocatalytic reactions, it is proved that there exists a global attractor and an inertial set in the L^2 phase space.